A Comparison of k-NN Methods for Time Series Classification and Regression

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Abstract. Interpretability is as important in data analytics on time series data as in other areas of analytics. For this reason, k-Nearest Neighbour methods have an important part to play; however coming up with effective similarity/distance measures is not straightforward given the nature of time series data. In this paper we survey the state-of-the-art distance measures used for time series analysis and evaluate their effectiveness on sample regression and classification tasks. In some circumstances, a 'global' method such as Dynamic Time Warping is effective, whereas for other datasets 'feature-based' methods work better.

Keywords: Regression, Classification, Time Series Data

1 Introduction

k-Nearest Neighbour (k-NN) methods have a special status in data analytics because of the importance of explanation and insight. Because k-NN methods are transparent they produce models that are interpretable. This is also true in time series analysis [9] where side-by-side comparisons of time series can reveal similarities and differences between process. However, when using k-NN methods for time series analysis there is the added challenge of coming up with measures that truly capturing the similarity between time series. Two time series might still be similar if one is *stretched* or *displaced* in time with respect to the other. It may also be the case that similarity depends on short or even tiny *signatures* in the time series.

In this paper we provide a short survey of recent methods that address these challenges. The methods we consider are:

- **Dynamic Time Warping (DTW):** Because two time series may be fundamentally similar but offset or slightly distorted, DTW allows the time axis to be *warped* to identify underlying similarities [8].
- **Symbolic Aggregate Approximation (SAX):** The idea with SAX is to discretize the time series so that it can be represented as a sequence of symbols [10]. This allows the panoply of data mining methods for sequences to be applied on the sequence representation of the time series.

– **Symbolic Fourier Approximation (SFA):** SFA is like SAX except the sequence representation is produced from a discrete Fourier transform representation of the signal rather than a discretization of the signal itself. So SFA is a frequency domain rather than a time domain representation of the signal [12].

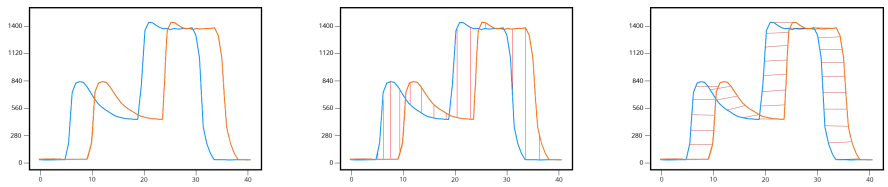
These three methods are described in detail in the next section. Then an evaluation of the performance of these methods on time series classification and regression tasks is presented in section 3. The evaluation also covers the thorny issue of parameter selection associated with these methods.

2 State-of-the-art Methods

In this section, we discuss the three methods (DTW, SAX, and SFA) covered in our analysis. The three methods we consider, all require parameter tuning. A hold-out test strategy is emplyed, where all parameter tuning is done through cross-validation on the training set before testing on the hold-out set. The parameters chosen are also presented in this section.

2.1 Dynamic Time Warping

To find the distance between two data series, the Euclidean distance formula is an obvious choice. But when dealing with time series data where the series may be displaced in time, the Euclidean distance may be large when the two series are similar, just off slightly on the time line (see Figure 1(a)). To tackle this situation Dynamic Time Warping offers us the flexibility of mapping the two data series in a non-linear fashion by warping the time axis [8]. It creates a cost matrix where the cells contain the distance value of the corresponding data-points and then finds the shortest path through the grid, which minimizes the total distance between them.



(a) Two time series displaced in time. (b) Mapping of data-points without warping. (c) Mapping of data-points with warping.

Fig. 1: Dynamic Time Warping.

Initially, when we restrict our algorithm with no warping allowed, the data points are linearly mapped between the two data series based on the common

time axis value. As seen in Figure 1(b), the algorithm does a poor job of matching the time series. But when we grant the DTW algorithm the flexibility of considering a warping window, the algorithm performs remarkably when mapping the data-points following the trend of the time series data, which can be visualized in Figure 1(c).

2.2 Symbolic Aggregate Approximation

Several symbolic representations of a time series data have been developed in recent decades with the objective of bringing the power of text processing algorithms to bear on time series problems. Keogh et *al.* provide an overview of these methods in their 2003 paper [10].

Symbolic Aggregate Approximation (SAX) is one such algorithm that achieves dimensionality and numerosity reduction and provides a distance measure that is a lower bound on corresponding distance measures on the orignal series [10]. In this case numerosity reduction refers to a more compact representation of the original data.

Piecewise Aggregate Approximation SAX uses Piecewise Aggregate Approximation (PAA) in its algorithm for dimensionality reduction. The fundamental idea behind the algorithm is to reduce the dimensionality of a time series by slicing it into equal-sized fragments which are then represented by the average of the values in the fragment.

PAA approximates a time series X of length n into vector $\bar{X} = (\bar{x}_1, \bar{x}_2, ..., \bar{x}_m)$ of any arbitrary length $m \leqslant n$, where each of x_i is computed as follows:

$$\bar{x}_i = \frac{m}{n} \sum_{j=\frac{n}{m}(i-1)+1}^{\frac{n}{m}i} x_j \tag{1}$$

This simply means that in order to reduce the size from n to m, the original time series is first divided into m fragments of equal size and then the mean values for each of these fragments are computed. The series constructed from these mean values is the PAA approximation of the original time series. There are two cases worth noting when using PAA. When $m = n$ the transformed representation is alike to the original input, and when $m = 1$ the transformed representation is just the mean of the original series [7]. Before the transformation of original data into the PAA representation, SAX also normalizes each of the time series to have a mean of zero and a standard deviation of one, given the difficulty of comparing time series of different scales [10, 6].

After the PAA transformation of the time series data, the output goes through another discretization procedure to obtain a discrete representation of the series. The objective is to discretize these levels into a bins of roughly equal size. These levels will typically follow a Gaussian distribution so these bins will get larger

(a) Raw time series data. (b) Time series after PAA dimensionality reduction. (c) Mapping of discretized bins to symbols in SAX.

Fig. 2: Symbolic Aggregate Approximation.

away from the mean. The breakpoints separating these discretized bins form a sorted list $B = \beta_1, ..., \beta_{a-1}$, such that the area under a $N(0, 1)$ Gaussian curve from β_i to $\beta_{i+1} = \frac{1}{a}$. β_0 and β_a are defined as $-\infty$ and ∞ respectively [10].

When all the breakpoints are computed, the original time series is discretized as follows. First, the PAA transformation of the time series is performed. Then each of the PAA coefficients less than the smallest breakpoint β_1 is mapped to the symbol s_1, and all coefficients between breakpoints β_1 and β_2 (second smallest breakpoint) are mapped to the symbol s_2, and so on, until the last PAA coefficient gets mapped. Here, s_1 and s_2 belongs to a set of symbols $S = (s_1, s_2, ..., s_m)$ to which the time series is mapped by SAX, where m is the size of symbol pool.

SAX also has a sliding window implemented in its algorithm, the size of which can be adjusted. It extracts the symbols present in that window frame and creates a word, which is just the concatenated sequence of symbols in that frame. This sliding window is then shifted to the right and another word is extracted corresponding to the new frame. This goes on until the window hits the end of the time series, and we get a "bag-of-words" representing that time series.

Once the data is converted to this symbolic representation, one can use this bag-of-words representation for calculating the distance between two time series using a string distance metric such as Levenshtein distance [15].

2.3 Symbolic Fourier Approximation

SFA was introduced by Schäfer et *al.* in 2012 as an alternative method to SAX built upon the idea of dimensionality reduction by symbolic representation. Unlike SAX which works on the time domain, SFA works on the frequency domain. In the frequency domain, each dimension contains approximate information about the whole time series. By increasing the dimensionality one can add detail, thus improving the overall quality of the approximation. In the time domain, we have to decide on a length of the approximation in advance and a prefix of this length only represents a subset of the time series [12].

Discrete Fourier Transform In contrast to SAX which uses PAA as its dimensionality reduction technique SFA, focusing on the frequency domain, uses the Discrete Fourier Transform (DFT). DFT is the equivalent of the continuous Fourier Transform for signals known only at N instants by sample times T, which is a finite series of data.

Let $X(t)$ be the continuous signal which is the source of the data. Let N samples be denoted $x[0], x[1], ..., x[N-1]$. The Fourier Transform of the original signal, $X(t)$, would be:

$$F(\omega_k) \triangleq \sum_{n=0}^{N-1} x(t_n) e^{-j\omega_k t_n}, k = 0, 1, 2, ..., N-1 \tag{2}$$

Simply stated, DFT analyzes a time domain signal $x(n)$ to determine the signal's frequency content $X[k]$. This is achieved by comparing $x[n]$ against signals known as sinusoidal basis functions, using correlation. The first few basis functions correspond to gradually changing regions and describe the coarse distribution, while later basis functions describe rapid changes like gaps or noise. Thus employing only the first few basis functions yields a good approximation of the time series[12].

The DFT Approximation is a part of the preprocessing step of SFA algorithm, where all time series data are approximated by computing DFT coefficients. When all these DFT coefficients are calculated, multiple discretisations are determined from all these DFT approximations using Multiple Coefficient Binning (MCB) as shown in Figure 3. MCB helps in mapping the coefficients to their symbols, and concatenates it to form an SFA word. Thus, this converts the time series into its symbolic representation.

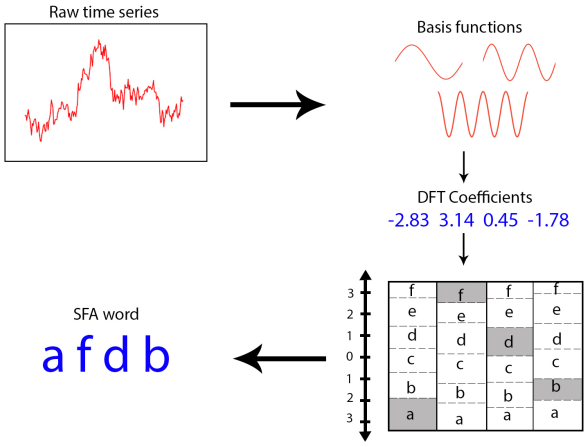


Fig. 3: Symbolic Fourier Approximation

As in SAX, there is a sliding window present here which serves the same purpose of extracting a word representing the data in that frame. Thus, the output of SFA for a given source time series is a bag-of-words symbolically representing the entire series in lower dimension.

3 Evaluation

When it comes to distance measures, Euclidean distance is a popular choice. It is easily implemented and has linear time complexity. Moreover, it is parameter-free, and is competitive against more complex approaches, notably when working on a large dataset. Nevertheless, Euclidean distance is very sensitive to noise and data being out of phase or warped in time, because of fixed mapping of the points of two time series[3].

Another distance measure worth considering is the Levenshtein distance, also popularly known as Edit distance, when working on the string or symbolic representation of the time series. Edit distance measures the distance between two strings, by computing the number of edits (insertion, deletion or substitution) required to match the strings. The Edit Distance on Real sequences (EDR)[1] and/or with real penalties has been shown to be very effective in time series classification and regression[3].

The Wagner-Fischer algorithm [14] has been shown to be an effective method for modelling penalties. It allows for different penalties for insertion, deletion and substitution and for distances within the alphabet to be included in the penalty score. For example, the basic implementation of Edit distance measures the distance between "boat" and "coat" as 1, and the distance between "coat" and "goat" is also computed as 1, because these strings are only 1 edit away. Whereas, the Wagner-Fischer algorithm with our assigned penalties, measures the distance between "coat" and "goat" as 4 because 'g' is 4 step away from 'c' in the alphabet.

We compare the three methods (DTW, SAX and SFA) used in time series classification or regression in a k-Nearest Neighbour model against the baselines of k-NN with Euclidean distance, and random 5-NN. Note that, SAX and SFA produces a symbolic representation of the time series, so we use EDR and WF as the underlying distance metric.

The evaluation of these methods was conducted on three separate datasets (details below). One of the datasets contains ordered classes so it is included in both the regression and classification evaluations.

3.1 The Data

In the evaluation we consider one classification and two regression problems.

- **Proximal Phalanx TW** This is one of the standard datasets from the UCR collection [2]. It is normally considered a classification task where the classes are ordered age categories but it is also treated as a regression problem here.

- **Jump Height** It is a jump exercise dataset where the time series was captured using a Shimmer sensor and the target variable is jump height.
- **Jump Class** This is a classification problem that is a superset of the Jump Height dataset. It contains three classes, one "correct" class (the Jump Height samples) and two incorrect classes.

Jumping Data The Jump Height and Jump Class datasets come from the same study. Ten participants (3 females, 7 males, age: 26.6 ±2.2, weight: 80.1±7.4 kg , height: 1.8±0.1 m) were recruited for this case-study. The Human Research Ethics Committee at University College Dublin approved the study protocol and written informed consent was obtained from all participants before testing. Participants did not have a current or recent musculoskeletal injury that would impair performance of Countermovement Jumps (CMJs) . Participants were equipped with a Shimmer 3 inertial measurement unit (IMU) [1] on their dominant foot. The IMU was configured to stream wide-range, tri-axial accelerometer data (±16 g) at 1024 Hz. AMTI Force-plate data [2] was also collected concurrently at 1000 Hz. This was used as a gold-standard tool to derive time in the air for each jump [4]. Each participant completed 20 CMJs with acceptable form and 40 with incorrect form (2 categories). The resulting data set consisted of 600 files of IMU data, 200 with acceptable form. This gives us a three class classification problem (600 samples) and a regression problem (200 samples).

The length of the IMU signals in each file ranged from 1231 to 6710 samples. The time-series used was the acceleration magnitude, derived from the accelerometer *x, y* and *z* signals whereby:

$$A_m = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

The target variable for the regression problem was time in the air in seconds, derived from the gold-standard force-plate data.

Note: Each sample in the datasets mentioned above consists of a single univariate time-series on which the models are trained to predict its target label. The unequal length of time-series was tackled by padding with 0's at the end of the shorter time-series when a pairwise distance was computed, between two time-series.

3.2 Parameter Selection

Parameter selection can have a significant impact on performance in Machine Learning and this is particularly true for the algorithms considered here. Here is a brief account of the parameter selection required for each of the methods.

[1] `http://www.shimmersensing.com/products/shimmer3-imu-sensor`
[2] `https://www.amti.biz`

DTW As mentioned, DTW gives the flexibility of mapping two time-series data warped in time in a non-linear fashion, with the help of a warping window. Thus, finding an optimal warping window is crucial for the algorithm. After cross-validation on the training data, we set the warping window for ProximalPhalanxTW, JumpingHeight and Jumping datasets to be 2, 26 and 26 respectively.

SAX The implementation of SAX [13] employed in this study has a large parameter space, consisting of five different parameters in total. The following are the parameters available:

1. Window size: The window size refers to the sliding window size w, which is the length of the frame on the x-axis. As SAX only considers the PAA coefficients present in the frame to map to symbols and create a SAX word, the window size plays a crucial role. The window size also affects the length of the words in the bag. It was set to 15, 40, and 30 for ProximalPhalanx, JumpHeight, and Jumping respectively.
2. PAA size: This parameter adjusts the number of discretized bins of equal-width on the x-axis. The mean of the data-points in each bin is calculated to compute the PAA coefficient. As PAA is the underlying dimensionality reduction method used in SAX, this parameter also requires a precise adjustment to describe the time-series well, with minimal loss of information. The PAA size was also set to be 15, 40 and 30 respectively for the three datasets.
3. Alphabet size: The discretization performed on the y-axis is governed by this parameter. Therefore, it is the number of bins of equal-probability denoted by a symbol from the Symbolic pool. For example, an alphabet size of 3 creates three levels or bins on the y-axis, and each of those bins can be represented by symbols, 'a','b' and 'c' respectively. It was set to of 5, 20 and 20 respectively.
4. NR Strategy: This parameter determines the numerosity reduction strategy applied to the converted SAX words to eliminate duplicate data. The three strategies which can be selected are "none", "exact" and "mindist". In all cases "none" was used.
5. Z-threshold: The time-series passed to PAA is normalised using z-normalisation. This parameter helps to adjust the threshold value of z-normalisation step in the algorithm. On preliminary inspection which resulted in 0.1 to be an optimum value for this parameter over the datasets, we set it to 0.1 for all SAX models in our experiments.

Note that as the parameter search space is high, we opted on setting the NR-strategy and z-threshold to be static on an optimum value following our preliminary experiments. The most influential parameters to this model are sliding window size, PAA size and alphabet size.

SFA The implementation of SFA [5] employed is a Python port of the original project[11]. There are five parameters to estimate which like in SAX, results in a large parameter space.

1. Window-size: As in SAX, the window size here also refers to the sliding window size w. The DFT coefficients present are mapped to symbols to create an SFA word. It was set to 5, 45, and 45 for ProximalPhalanx, JumpHeight, and Jumping datasets respectively.
2. Word-length: This parameter controls the length of the words in the bag-of-words. This parameter was set to be the same as the sliding-window size, i.e. 5, 45, and 45.
3. Alphabet-size: This serves the same purpose as in SAX. It is the quantization over the y-axis. This parameter was set to be 5, 20, and 20 for the three datasets.
4. Norm-mean: The data can be normalised to the signal mean or left unnormalised.
5. Lower-bounding: SFA uses lower-bounding distance measure during discretization with MCB [12] to have a reduced but still effective search space. It can be set to True or False, depending on the application if this feature is wanted.

The parameter search space is high as in SAX, we opted on setting the Norm-mean and Lower-bounding parameters to be True following our preliminary experiments. The most influential parameters to this model are sliding window size, word length and alphabet size.

Note k-NN has only one parameter, the neighbourhood size (k). We chose to go with the best k value for each model by measuring its performance by 10-fold cross-validation on the train set.

3.3 Model Performance

This section deals with the performance of the three methods, DTW, SAX, and SFA against two baseline models, Euclidean distance (ED) and Random 5-NN. For SAX and SFA we consider both EDR and WF string distance measures so we evaluate five candidates against two baselines.

Figure 4 illustrates the performance of the models on time-series regression task over two datasets. Mean Absolute Percentage Error (MAPE) metric was employed to evaluate the performance, where a lower MAPE score signifies better performance. While DTW does not perform well on the Jump Height task it is the best model on the Proximal Phalanx task. Here, we can see that SFA with WF is consistent than the other models, showing robustness.

Figure 5 shows the performance on the classification tasks. Here the measure is classification accuracy so higher is better. When it comes to time-series classification we can see DTW performed best, and SFA-WF comes a close second. The WF string distance measure shows a significant impact when working on the Jumping dataset.

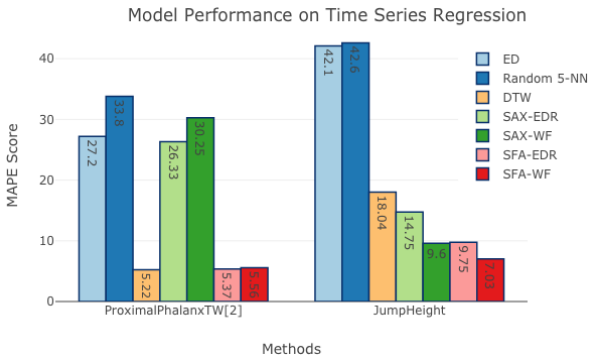


Fig. 4: Performance of various k-NN methods over two datasets on time series regression task. The evaluation measure is Mean Absolute Percentage Error (MAPE), where lower is better.

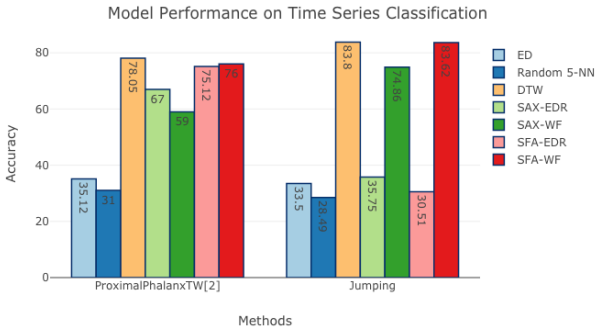


Fig. 5: Performance of various k-NN methods on the classification task. The evaluation measure is Accuracy, where higher is better.

One overall conclusion we can draw is that SFA-WF always performs well. DTW does well except in the Jump Height regression task. We propose that this is due to the global nature of DTW which seeks a mapping across the whole time series. This is less appropriate in the Jump Height task where sub-regions of the time series are particularly important in predicting the outcome.

4 Conclusions & Future Work

When it comes to time-series regression and classification, k-NN-DTW is one of the most robust and well-performing methods on a wide array of datasets. SFA performs significantly better than SAX due to the fact, that SFA builds upon DFT, which is significantly more accurate than the PAA dimensionality reduction technique SAX employs[12]. The characteristic of the time series present

in the datasets has a direct impact on the variations of the results obtained. In the regression tasks, if all of the time series is important (as in the ProximalPhalanxTW[2] dataset) DTW performs better than the other methods. Whereas, when considering datasets (e.g. JumpHeight) having time-series with importance at its feature or sub-sequence level, 'feature-based' methods like SAX and SFA performs significantly better. It is worth noting that various other text-mining algorithms can be incorporated over the 'bag-of-words' representation of a time-series data as a feature selection process. Having such a process could potentially improve the performance of SAX and SFA, in removing less informative features.

Acknowledgments

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References

1. Chen, L., Özsu, M.T., Oria, V.: Robust and fast similarity search for moving object trajectories. Proceedings of the 2005 ACM SIGMOD international conference on Management of data - SIGMOD 05 (2005)
2. Chen, Y., Keogh, E., Hu, B., Begum, N., Bagnall, A., Mueen, A., Batista, G.: The UCR time series classification archive (July 2015), `www.cs.ucr.edu/~eamonn/time_series_data/`
3. Ding, H., Trajcevski, G., Scheuermann, P., Wang, X., Keogh, E.: Querying and mining of time series data. Proceedings of the VLDB Endowment 1(2), 15421552 (Jan 2008)
4. Glatthorn, J.F., Gouge, S., Nussbaumer, S., Stauffacher, S., Impellizzeri, F.M., Maffiuletti, N.A.: Validity and reliability of optojump photoelectric cells for estimating vertical jump height. The Journal of Strength & Conditioning Research 25(2), 556–560 (2011)
5. Harford, S., Darabi, H.: Sfa_python. `https://github.com/sharford5/SFA_Python` (2017)
6. Keogh, E., Kasetty, S.: On the need for time series data mining benchmarks. Proceedings of the eighth ACM SIGKDD international conference on Knowledge discovery and data mining - KDD 02 (2002)
7. Keogh, E.J., Pazzani, M.J.: Scaling up dynamic time warping for datamining applications. Proceedings of the sixth ACM SIGKDD international conference on Knowledge discovery and data mining - KDD 00 (2000)
8. Keogh, E.J., Pazzani, M.J.: Derivative dynamic time warping. In: Proceedings of the 2001 SIAM International Conference on Data Mining. pp. 1–11. SIAM (2001)
9. Le Nguyen, T., Gsponer, S., Ifrim, G.: Time series classification by sequence learning in all-subsequence space. In: Data Engineering (ICDE), 2017 IEEE 33rd International Conference on. pp. 947–958. IEEE (2017)
10. Lin, J., Keogh, E., Lonardi, S., Chiu, B.: A symbolic representation of time series, with implications for streaming algorithms. Proceedings of the 8th ACM SIGMOD workshop on Research issues in data mining and knowledge discovery - DMKD 03 (2003)

11. Schäfer, P.: Sfa. `https://github.com/patrickzib/SFA` (2015)
12. Schäfer, P., Högqvist, M.: SFA: a symbolic fourier approximation and index for similarity search in high dimensional datasets. In: Proceedings of the 15th International Conference on Extending Database Technology. pp. 516–527. ACM (2012)
13. Senin, P., Lin, J., Wang, X., Oates, T., Gandhi, S., Boedihardjo, A.P., Chen, C., Frankenstein, S., Lerner, M.: Grammarviz 2.0: A tool for grammar-based pattern discovery in time series. In: Calders, T., Esposito, F., Hüllermeier, E., Meo, R. (eds.) Machine Learning and Knowledge Discovery in Databases. pp. 468–472. Springer Berlin Heidelberg, Berlin, Heidelberg (2014)
14. Wagner, R.A., Fischer, M.J.: The string-to-string correction problem. Journal of the ACM 21(1), 168173 (Jan 1974)
15. Yujian, L., Bo, L.: A normalized levenshtein distance metric. IEEE transactions on pattern analysis and machine intelligence 29(6), 1091–1095 (2007)